

Dipion invariant mass distribution of the anomalous $\Upsilon(1S)\pi^+\pi^-$ and $\Upsilon(2S)\pi^+\pi^-$ production near the peak of $\Upsilon(10860)$

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Considering the defects of the previous work for estimating the anomalous production rates of $e^+e^- \rightarrow \Upsilon(1S)\pi^+\pi^-$, $\Upsilon(2S)\pi^+\pi^-$ near the peak of the $\Upsilon(5S)$ resonance at $\sqrt{s} = 10.87$ GeV [K.F. Chen *et al.* (Belle Collaboration), Phys. Rev. Lett. **100**, 112001 (2008)], we suggest a new scenario where the contributions from the direct dipion transition and the final state interactions interfere to result in not only the anomalously large production rates, but also the lineshapes of the differential widths consistent with the experimental measurement when assuming the reactions are due to the dipion emission of $\Upsilon(5S)$. At the end, we raise a new puzzle that the predicted differential width $d\Gamma(\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-)/d\cos\theta$ has a discrepant trend from the data while other predictions are well in accord with the data. It should be further clarified by more accurate measurements carried by future experiments.

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Looking at the spectra of the $b\bar{b}$ system listed in the particle data book [1], there exist six bottomonia with $J^{PC} = 1^{--}$, which are, respectively, $\Upsilon(nS)$ ($n=1,2,3,4$), $\Upsilon(10860)$, and $\Upsilon(11020)$. The first five resonances are orderly assigned as nS ($n = 1, \dots, 5$) $b\bar{b}$ states, whereas the extra one, $\Upsilon(11020)$, may be the $6S$ state. The estimate on their spectra in the potential model supports such assignments [2–4]. However, recently anomalous large rates of $e^+e^- \rightarrow \Upsilon(1S)\pi^+\pi^-$, $\Upsilon(2S)\pi^+\pi^-$ near the peak of the $\Upsilon(5S)$ resonance at $\sqrt{s} = 10.87$ GeV were observed by the Belle Collaboration to be larger than the dipion-transition rates between the lower members of the Υ family by 2 orders of magnitude. The Belle data are $\Gamma(\Upsilon(5S) \rightarrow \Upsilon(1S)\pi^+\pi^-) = 0.59 \pm 0.04(\text{stat}) \pm 0.09(\text{syst})$ MeV and $\Gamma(\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-) = 0.85 \pm 0.07(\text{stat}) \pm 0.16(\text{syst})$ MeV [5]. The Belle observation has stimulated theorists' extensive interest in exploring the reason that results in such anomalous phenomena.

There are two possibilities that may offer reasonable interpretations of the anomalous large rates. First, these anomalous production rates announced by Belle are from an exotic resonance structure different from $\Upsilon(10860)$. The second is that there may exist extra contributions that differ from the direct dipion emission $\Upsilon(10860) \rightarrow \Upsilon(1S, 2S)\pi^+\pi^-$. It is expected that with a careful analysis we may eventually identify the reasonable, or at least the dominant source of the large rates. Thus one not only obtains the branching ratios, but also needs to fit the lineshapes of the differential widths over the invariant mass of dipion and over the angular distribution $\cos\theta$.

Along the first route, Ali *et al.* suggested a tetraquark interpretation of $Y_b(10890) = [bq][\bar{b}\bar{q}]$ [6–8] and analyzed the Belle data [5] for the anomalous $\Upsilon(1S)\pi^+\pi^-$ and $\Upsilon(2S)\pi^+\pi^-$ productions near the $\Upsilon(5S)$ resonance. By fitting

the $\pi^+\pi^-$ invariant mass spectrum and the $\cos\theta$ distributions for $Y_b(10890) \rightarrow \Upsilon(nS)\pi^+\pi^-$ ($n = 1, 2$) shown in Figs. 1 and 2 of Ref. [7], they claimed that the tetraquark interpretation can well describe the anomalous rates of the two-pion-production. In their scenario, there are non-resonant and resonant contributions interfering to result in the branching ratio and differential widths. In that work [7, 8], a simple Lorentz structure Lagrangian was introduced to stand as the effective interaction.

An alternative scenario was proposed, namely, the final state interaction of $\Upsilon(10860)$ decaying into $\Upsilon(1S, 2S)\pi^+\pi^-$ can be realized via sub-processes $\Upsilon(10860) \rightarrow B^{(*)}\bar{B}^{(*)} \rightarrow \Upsilon(1S, 2S)\pi^+\pi^-$ [9–11]. It was also claimed that the anomalous $\Upsilon(1S)\pi^+\pi^-$ and $\Upsilon(2S)\pi^+\pi^-$ productions near the $\Upsilon(5S)$ resonance receive reasonable explanations. The tetraquark interpretation presented in Ref. [7] is evidently not unique, and, moreover below we will show that the dipion invariant mass distribution and the angular distribution of $Y_b(10890) \rightarrow \Upsilon(2S)\pi^+\pi^-$ obtained with the tetraquark picture proposed in Ref. [7] cannot explain the Belle data well.

Using the formulas and parameters given in Ref. [7] to fit the data given by the Belle Collaboration, we find obvious discrepancies. Namely, as shown in the following figure, employing the formulas and parameters given in Ref. [7], we obtain the solid lines for the dipion invariant mass distribution and angular distribution for $Y_b(10890) \rightarrow \Upsilon(2S)\pi^+\pi^-$ (see Fig. 1), and obviously they do not fit the data points which are marked in the figure. If, with the formulas given by the authors of Ref. [7], we fit the dipion invariant mass distribution (the dashed line) as the left-hand diagram of Fig. 1 to gain the model parameters, then applying the parameters to calculate the angular distribution $d\Gamma/d\cos\theta$, we would have the dashed line on the right-side of Fig. 1. Inversely, if we first fit the differential width $d\Gamma/d\cos\theta$ to fix the parameters (the dotted-line on the right-hand side of Fig. 1), using those parameters would result in the dotted-line on the left-hand side diagram of Fig. 1. All the results contradict the data; even the trend does not coincide. Therefore, it seems that the tetraquark scenario

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does not work well to some extent.

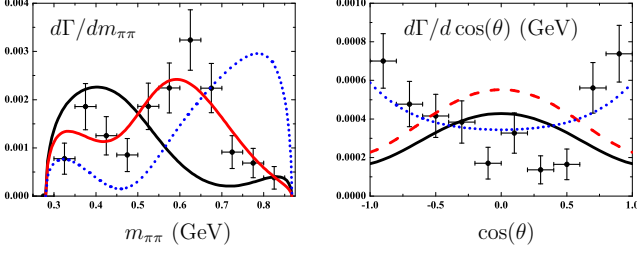


FIG. 1: (color online). Dipion invariant mass ($m_{\pi\pi}$) distribution (left-hand side) and the $\cos\theta$ distribution (right-hand side) of $\Upsilon(2S)\pi^+\pi^-$ production. The dots with error bars are the results measured by Belle. The solid lines denote the results reproduced with the parameters shown in Table. II of Ref. [7]. With the same formula as that in Ref. [7], we refit the experimental data. Here, the red dashed-line curves and blue dotted-line curves are the fitting results with parameters $\{F = 0.933 \pm 0.396, \beta = 0.692 \pm 0.202, f_\sigma = 9.405 \pm 2.409, \phi = -0.460 \pm 0.245 \text{ Rad}\}$ and $\{F = 1.056 \pm 1.348, \beta = 0.467 \pm 0.578, f_\sigma = 10.354 \pm 15.163, \phi = -1.785 \pm 1.223 \text{ Rad}\}$, respectively, which correspond to the best fits to the dipion invariant mass distribution and the $\cos\theta$ distribution, respectively.

In Ref. [9], the authors considered the sub-processes $\Upsilon(10860) \rightarrow B^{(*)}\bar{B}^{(*)} \rightarrow \Upsilon(1S, 2S)\pi^+\pi^-$ which are supposed to be the final state interaction of $\Upsilon(10860) \rightarrow \Upsilon(1S, 2S)\pi^+\pi^-$. They concluded that as the absorptive (imaginary) part of the triangle diagrams dominate, one can expect an enhancement of 200 ~ 600 times compared to the partial widths of dipion emission $\Upsilon(nS) \rightarrow \Upsilon(1S, 2S)\pi^+\pi^-$ ($n \leq 3$). Moreover, even though for $\Upsilon(4S)$, the $B\bar{B}$ channel is open, but due to the limit in phase space, the p -value suppresses the contribution from the $B\bar{B}$ intermediate states. The data show that the partial width of $\Upsilon(4S) \rightarrow \Upsilon(1S, 2S)\pi^+\pi^-$ is only 2~4 times larger than that of $\Upsilon(3S) \rightarrow \Upsilon(1S, 2S)\pi^+\pi^-$ by the Belle and Babar measurements. Thus it seems that the largeness of the dipion emission of $\Upsilon(10860)$ can be explained as coming from the final state interactions. However, in Ref. [9], the authors did not give a fit to the line shapes of the dipion invariant mass distribution $d\Gamma/dm_{\pi^+\pi^-}$ and the differential width $d\Gamma/d\cos\theta$ explicitly, so they claimed that their results were roughly consistent with data.

Even though the partial width of $\Upsilon(10860) \rightarrow \Upsilon(1S, 2S)\pi^+\pi^-$ is larger than that of $\Upsilon(nS) \rightarrow \Upsilon(mS)\pi^+\pi^-$ ($n = 3, 4, m = 1, 2$) by two orders, there is still no compelling reason to ignore the contribution from the direct dipion transition process. Especially an interference between the direct transition and the contribution from the final state interaction (intermediate heavy-meson loops) may result in a sizable change to each scenario. We notice that $\Gamma(\Upsilon(10860) \rightarrow \Upsilon(2S)\pi^+\pi^-) = 0.85 \pm 0.07(\text{stat}) \pm 0.16(\text{syst}) \text{ MeV}$ which is larger than $\Gamma(\Upsilon(10860) \rightarrow \Upsilon(1S)\pi^+\pi^-) = 0.59 \pm 0.04(\text{stat}) \pm 0.09(\text{syst}) \text{ MeV}$ [5], even though the later one has a larger final-state phase space. If the largeness is due to the composition of $\Upsilon(10860)$, this relation should be inverted. This intriguing experimental fact reported by Belle [5] seems to be evidence to support an interference between the direct transition and the contribution from the final state

interaction for $\Upsilon(10860) \rightarrow \Upsilon(1S, 2S)\pi^+\pi^-$. Obviously, the final state interaction that is realized via the heavy-meson loops is a simplified version of the complicated multichannel dynamics [11]. The direct dipion transition was dealt with in terms of the QCD multipole expansion method where there are two color-E1 transition and the two color fields eventually hadronize into two pions. Yan and Kuang [12] established the theoretical framework for the multipole expansion method, where the intermediate state between the two E1 transitions the quark pair $Q\bar{Q}$ resides in a color-octet where Q stands for heavy quark b or c . Here, we do not intend to calculate the contribution from the direct transition, but set it as an effective interaction with the free parameter, which will be fixed by fitting data.

In this work, we suggest that the total decay amplitude of $\Upsilon(5S) \rightarrow \Upsilon(1S, 2S)\pi^+\pi^-$ should include a few terms such as

$$\mathcal{M}_{\text{total}} = \mathcal{M} \left[\begin{array}{c} \Upsilon(5S) \\ \hline \text{---} \end{array} \begin{array}{c} \Upsilon(nS) \\ \pi^+ \\ \pi^- \end{array} \right] + \sum_R e^{i\phi_R^{(n)}} \mathcal{M} \left[\begin{array}{c} \Upsilon(5S) \\ \hline \text{---} \end{array} \begin{array}{c} \Upsilon(nS) \\ \pi^+ \\ \pi^- \end{array} \right]_R, \quad (1)$$

where we take into account contributions from different intermediate resonances R to dipion, i.e., $R = \{\sigma(600), f_0(980), f_2(1270)\}$ to the $\Upsilon(1S)\pi^+\pi^-$ channel and $R = \{\sigma(600), f_0(980)\}$ to the $\Upsilon(2S)\pi^+\pi^-$ channel, which are allowed by the phase spaces. The phase angles $\phi_R^{(n)}$ are introduced, which will be fixed in this paper.

In general, the decay amplitude of the direct production of $\Upsilon(5S) \rightarrow \Upsilon(1S, 2S)\pi^+\pi^-$ is expressed as

$$\begin{aligned} & \mathcal{M}[\Upsilon(5S) \rightarrow \Upsilon(nS)(p_1)\pi^+(p_2)\pi^-(p_3)] \\ &= \epsilon_{\Upsilon(5S)} \cdot \epsilon_{\Upsilon(nS)}^* \left\{ \left[q^2 - \kappa(\Delta M)^2 \left(1 + \frac{2m_\pi^2}{q^2} \right) \right]_{\text{S-wave}} \right. \\ & \quad \left. + \left[\frac{3}{2} \kappa(\Delta M)^2 - q^2 \right] \left(1 - \frac{4m_\pi^2}{q^2} \right) \left(\cos^2\theta - \frac{1}{3} \right) \right\}_{\text{D-wave}} \mathcal{A}, \end{aligned}$$

which was first written by Novikov and Shifman in Ref. [13] while studying $\psi' \rightarrow J/\psi\pi^+\pi^-$ decay, where the S-wave and D-wave contributions are distinguished by the subscripts S-wave and D-wave. ΔM denotes the mass difference between $\Upsilon(5S)$ and $\Upsilon(nS)$. $q^2 = (p_2 + p_3)^2 \equiv m_{\pi^+\pi^-}^2$ is the invariant mass of $\pi^+\pi^-$. θ is the angle between $\Upsilon(5S)$ and π^- in the $\pi^+\pi^-$ rest frame. In Ref. [7], Ali *et al.* also adopted the expression in Eq. (2) and introduced an extra form factor $\mathcal{A} = F/f_\pi^2$ with $f_\pi = 130 \text{ MeV}$.

As shown in Fig. 2, there are six diagrams corresponding to $\Upsilon(5S)$ decays into $\Upsilon(nS)S$ and $\Upsilon(nS)f_2(1270)$ respectively, and S and $f_0(980)$ eventually turn into two pions; thus, as on-shell intermediate states they contribute to $\Upsilon(5S) \rightarrow \Upsilon(nS)\pi^+\pi^-$. Such subsequent processes are attributed to the final state interaction. In this work, we adopt the effective Lagrangian approach to write out the decay amplitudes for the diagrams in Fig. 2, where the relevant Lagrangians include

(10)

$$\mathcal{L}_{\Upsilon\mathcal{B}\mathcal{B}} = ig_{\Upsilon\mathcal{B}\mathcal{B}}\Upsilon_\mu(\partial^\mu\mathcal{B}\mathcal{B}^\dagger - \mathcal{B}\partial^\mu\mathcal{B}^\dagger), \quad (2)$$

$$\mathcal{L}_{\Upsilon\mathcal{B}^*\mathcal{B}} = -ig_{\Upsilon\mathcal{B}^*\mathcal{B}}\epsilon^{\mu\nu\alpha\beta}\partial_\mu\Upsilon_\nu(\partial_\alpha\mathcal{B}_\beta^*\mathcal{B}^\dagger + \mathcal{B}\partial_\alpha\mathcal{B}_\beta^{*\dagger}), \quad (3)$$

$$\begin{aligned} \mathcal{L}_{\Upsilon\mathcal{B}^*\mathcal{B}^*} = & -ig_{\Upsilon\mathcal{B}^*\mathcal{B}^*}\{\Upsilon^\mu(\partial_\mu\mathcal{B}^{*\nu}\mathcal{B}_\nu^{*\dagger} - \mathcal{B}^{*\nu}\partial_\mu\mathcal{B}_\nu^{*\dagger}) \\ & + (\partial_\mu\Upsilon_\nu\mathcal{B}^{*\nu} - \Upsilon_\nu\partial_\mu\mathcal{B}^{*\nu})\mathcal{B}^{*\mu\dagger} \\ & + \mathcal{B}^{*\mu}(\Upsilon^\nu\partial_\mu\mathcal{B}_\nu^{*\dagger} - \partial_\mu\Upsilon_\nu\mathcal{B}^{*\nu\dagger})\}, \end{aligned} \quad (4)$$

and

$$\mathcal{L}_{S\mathcal{B}^{(*)}\mathcal{B}^{(*)}} = g_{S\mathcal{B}S}\mathcal{S}\mathcal{B}\mathcal{B}^\dagger - g_{S^*\mathcal{B}^*S}\mathcal{S}\mathcal{B}^*\mathcal{B}^{*\dagger} \quad (5)$$

where $\mathcal{B} = (\bar{B}^0, B^-, B_s^-)$ and $(\mathcal{B}^\dagger)^T = (B^0, B^+, B_s^+)$. Thus, the decay amplitudes corresponding to Figs. 2(a)-2(f) are expressed as

$$\begin{aligned} \mathcal{M}_a = & (i)^3 \int \frac{d^4q}{(2\pi)^4} [ig_{\Upsilon(5S)BB}\epsilon_{\Upsilon(5S)}^\mu (ip_{2\mu} - ip_{1\mu})] \\ & \times [ig_{\Upsilon(nS)BB}\epsilon_{\Upsilon(nS)}^\rho (-ip_{1\rho} - iq_\rho)] [g_{BBS}] \\ & \times \frac{1}{p_1^2 - m_B^2} \frac{1}{p_2^2 - m_B^2} \frac{1}{q^2 - m_B^2} \mathcal{F}(q^2), \end{aligned} \quad (6)$$

$$\begin{aligned} \mathcal{M}_b = & (i)^3 \int \frac{d^4q}{(2\pi)^4} [-g_{\Upsilon(5S)BB}\epsilon_{\mu\nu\alpha\beta}(-ip_0^\mu)\epsilon_{\Upsilon(5S)}^\nu(ip_2^\alpha)] \\ & \times [-g_{\Upsilon(nS)BB}\epsilon_{\delta\tau\theta\phi}(ip_3^\delta)\epsilon_{\Upsilon(nS)}^\tau(iq^\theta)] [-g_{B^*B^*S}] \\ & \times \frac{1}{p_1^2 - m_B^2} \frac{-g^{\beta\rho} + p_2^\beta p_2^\rho/m_{B^*}^2}{p_2^2 - m_{B^*}^2} \frac{-g^{\phi\rho} + q^\phi q^\rho/m_{B^*}^2}{q^2 - m_{B^*}^2} \mathcal{F}(q^2), \end{aligned} \quad (7)$$

$$\begin{aligned} \mathcal{M}_c = & (i)^3 \int \frac{d^4q}{(2\pi)^4} [-g_{\Upsilon(5S)B^*B}\epsilon_{\mu\nu\alpha\beta}(-ip_0^\mu)\epsilon_{\Upsilon(5S)}^\nu(ip_1^\alpha)] \\ & \times [-g_{\Upsilon(nS)B^*B}\epsilon_{\delta\tau\theta\phi}(ip_3^\delta)\epsilon_{\Upsilon(nS)}^\tau(-ip_1^\theta)] [g_{BBS}] \\ & \times \frac{-g^{\beta\phi} + p_1^\beta p_1^\phi/m_{B^*}^2}{p_1^2 - m_{B^*}^2} \frac{1}{p_2^2 - m_B^2} \frac{1}{q^2 - m_B^2} \mathcal{F}(q^2), \end{aligned} \quad (8)$$

$$\begin{aligned} \mathcal{M}_d = & (i)^3 \int \frac{d^4q}{(2\pi)^4} [-ig_{\Upsilon(5S)B^*B^*}\epsilon_{\Upsilon(5S)}^\mu ((ip_{2\mu} - ip_{1\mu})g_{\nu\rho} \\ & + (-ip_{0\rho} - ip_{2\rho})g_{\mu\nu} + (ip_{1\nu} + ip_{0\nu})g_{\mu\rho})] \\ & \times [-ig_{\Upsilon(nS)B^*B^*}\epsilon_{\Upsilon(nS)}^\phi ((-ip_{1\phi} - iq_\phi)g_{\alpha\beta} \\ & + (ip_{3\beta} + ip_{1\beta})g_{\alpha\phi} + (iq_\alpha - ip_{3\alpha})g_{\beta\phi})] [-g_{B^*B^*S}] \\ & \times \frac{-g^{\rho\alpha} + p_1^\rho p_1^\alpha/m_{B^*}^2 - g^{\nu\tau} + p_2^\nu p_2^\tau/m_{B^*}^2}{p_1^2 - m_{B^*}^2} \frac{-g^{\phi\tau} + p_2^\phi p_2^\tau/m_{B^*}^2}{p_2^2 - m_{B^*}^2} \\ & \times \frac{-g^{\beta\tau} + q^\beta q^\tau/m_{B^*}^2}{q^2 - m_{B^*}^2} \mathcal{F}(q^2), \end{aligned} \quad (9)$$

$$\begin{aligned} \mathcal{M}_e = & (i)^3 \int \frac{d^4q}{(2\pi)^4} [-g_{\Upsilon(5S)BB}\epsilon_{\mu\nu\alpha\beta}(-ip_0^\mu)\epsilon_{\Upsilon(5S)}^\nu(ip_2^\alpha)] \\ & \times [-g_{\Upsilon(nS)BB}\epsilon_{\delta\tau\theta\phi}(ip_3^\delta)\epsilon_{\Upsilon(nS)}^\tau(iq^\theta)] \\ & \times [g_{f_2B^*B^*}\epsilon_{f_2}^{\rho\lambda}(g_{\rho\kappa}g_{\lambda\gamma} + g_{\rho\gamma}g_{\lambda\kappa} - g_{\rho\lambda}g_{\gamma\kappa})] \\ & \times \frac{1}{p_1^2 - m_B^2} \frac{-g^{\beta\rho} + p_2^\beta p_2^\rho/m_{B^*}^2}{p_2^2 - m_{B^*}^2} \frac{-g^{\phi\rho} + q^\phi q^\rho/m_{B^*}^2}{q^2 - m_{B^*}^2} \mathcal{F}(q^2), \end{aligned}$$

$$\begin{aligned} \mathcal{M}_f = & (i)^3 \int \frac{d^4q}{(2\pi)^4} [-ig_{\Upsilon(5S)B^*B^*}\epsilon_{\Upsilon(5S)}^\mu ((ip_{2\mu} - ip_{1\mu})g_{\nu\rho} \\ & + (-ip_{0\rho} - ip_{2\rho})g_{\mu\nu} + (ip_{1\nu} + ip_{0\nu})g_{\mu\rho})] \\ & \times [-ig_{\Upsilon(nS)B^*B^*}\epsilon_{\Upsilon(nS)}^\phi ((-ip_{1\phi} - iq_\phi)g_{\alpha\beta} \\ & + (ip_{3\beta} + ip_{1\beta})g_{\alpha\phi} + (iq_\alpha - ip_{3\alpha})g_{\beta\phi})] \\ & \times [g_{f_2B^*B^*}\epsilon_{f_2}^{\alpha\beta}(g_{\alpha\kappa}g_{\beta\gamma} + g_{\alpha\gamma}g_{\beta\kappa} - g_{\alpha\beta}g_{\gamma\kappa})] \\ & \times \frac{-g^{\rho\alpha} + p_1^\rho p_1^\alpha/m_{B^*}^2 - g^{\nu\tau} + p_2^\nu p_2^\tau/m_{B^*}^2}{p_1^2 - m_{B^*}^2} \frac{-g^{\phi\tau} + p_2^\phi p_2^\tau/m_{B^*}^2}{p_2^2 - m_{B^*}^2} \\ & \times \frac{-g^{\beta\tau} + q^\beta q^\tau/m_{B^*}^2}{q^2 - m_{B^*}^2} \mathcal{F}(q^2). \end{aligned} \quad (11)$$

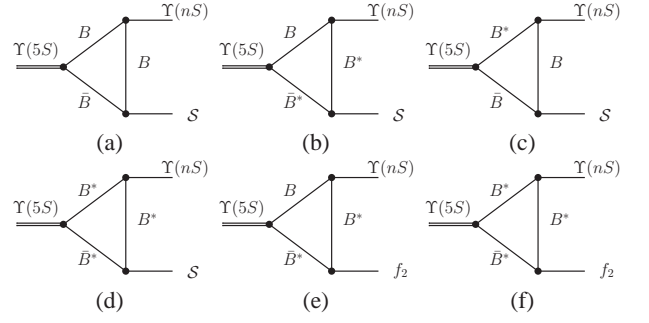


FIG. 2: The schematic diagrams for $\Upsilon(5S)$ decays into $\Upsilon(nS)S$ and $\Upsilon(nS)f_2(1270)$ ($n = 1, 2$) via $B^{(*)}$ meson loops.

With above preparation, the amplitudes of $\Upsilon(5S) \rightarrow \Upsilon(nS)\pi^+\pi^-$ via the re-scattering can be parameterized as

$$\begin{aligned} \mathcal{M}[\Upsilon(5S) \rightarrow B^{(*)}\bar{B}^{(*)} \rightarrow \Upsilon(nS)(p_1)\pi^+(p_2)\pi^-(p_3)]_S \\ = \left\{ g_{0S}^{(n)} g_{\mu\nu} p_1 \cdot q + g_{0D}^{(n)} p_{1\mu} q_\nu \right\} \frac{\epsilon_{\Upsilon(5S)}^\mu \epsilon_{\Upsilon(nS)}^{*\nu} g_{S\pi\pi} p_2 \cdot p_3}{q^2 - m_S^2 + im_S \Gamma_S}, \end{aligned} \quad (12)$$

$$\begin{aligned} \mathcal{M}[\Upsilon(5S) \rightarrow B^{(*)}\bar{B}^{(*)} \rightarrow \Upsilon(nS)(p_1)\pi^+(p_2)\pi^-(p_3)]_{f_2(1270)} \\ = \left\{ g_{2S}^{(n)} [g_{\mu\rho} g_{\nu\lambda} + g_{\mu\lambda} g_{\nu\rho}] (p_1 \cdot q)^2 + [g_{2D_1}^{(n)} g_{\mu\nu} p_{1\rho} q_\lambda \right. \\ \left. + g_{2D_2}^{(n)} (g_{\mu\rho} q_\nu p_{1\lambda} + g_{\mu\lambda} q_\nu p_{1\rho}) + g_{2D_3}^{(n)} (g_{\nu\lambda} q_\mu p_{1\rho} \right. \\ \left. + g_{\nu\rho} q_\mu p_{1\lambda})] p_1 \cdot q + g_{2G}^{(n)} q_\mu q_\nu p_{1\rho} p_{1\lambda} \right\} \frac{\epsilon_{\Upsilon(5S)}^\mu \epsilon_{\Upsilon(nS)}^{*\nu} \mathcal{P}_{f_2}^{\rho\lambda\alpha\beta}(q)}{q^2 - m_{f_2}^2 + im_{f_2} \Gamma_{f_2}} \\ \times g_{f_2\pi\pi} p_{2\alpha} p_{2\beta}, \end{aligned} \quad (13)$$

corresponding to the contributions from the intermediate scalar states $S = \{\sigma(600), f_0(980)\}$ and the tensor meson $f_2(1270)$. In the above equation, $\mathcal{P}_{f_2}^{\rho\lambda\alpha\beta}(q)$ is defined as

$$\mathcal{P}_{f_2}^{\rho\lambda\alpha\beta}(q) = \frac{1}{2}(\tilde{g}^{\rho\alpha}\tilde{g}^{\lambda\beta} + \tilde{g}^{\rho\beta}\tilde{g}^{\lambda\alpha}) - \frac{1}{3}\tilde{g}^{\rho\lambda}\tilde{g}^{\alpha\beta}$$

with $\tilde{g}^{\alpha\beta} = g^{\alpha\beta} - q^\alpha q^\beta/m_{f_2}^2$. Then the differential decay width reads as

$$d\Gamma = \frac{1}{3} \frac{1}{(2\pi)^3} \frac{1}{32M_{\Upsilon(5S)}^3} |\mathcal{M}|_{\text{total}}^2 dm_{\Upsilon\pi}^2 dm_{\pi\pi}^2, \quad (14)$$

TABLE I: The resonance parameters (in units of GeV) used in this work [1, 15].

| | | | | | |
|-------------|--------|----------------|-------|---------------------|-------|
| $m_{Y(5S)}$ | 10.870 | m_{σ} | 0.526 | Γ_{σ} | 0.302 |
| $m_{Y(1S)}$ | 9.460 | $m_{f_0(980)}$ | 0.980 | $\Gamma_{f_0(980)}$ | 0.070 |
| $m_{Y(2S)}$ | 10.024 | m_{f_2} | 1.275 | Γ_{f_2} | 0.185 |

where $m_{\pi\pi}^2 = (p_1 + p_2)^2$ and $m_{\pi\pi}^2 = (p_2 + p_3)^2$. The factor $1/3$ comes from an average over the polarizations of the initial $Y(5S)$ state and in Ref. [7], this factor was missing.

For the re-scattering process, the effective Lagrangian for coupling bottomonia to the bottomed mesons is determined based on the heavy quark effective theory [14]. The coupling constants for $Y(5S)B^{(*)}B^{(*)}$ are evaluated by fitting the partial decay widths while for $Y(nS)B^{(*)}B^{(*)}$ ($n = 1, 2$) and $SB^{(*)}B^{(*)}$, the coupling constants are directly taken from Ref. [9]. In the re-scattering picture, for the $Y(5S) \rightarrow Y(1S)\pi^+\pi^-$ process, the tensor meson $f_2(1270)$ should be included. Compared to the S -wave coupling $f_2B^*B^*$, the D -wave couplings f_2BB and f_2BB^* are negligible due to the high partial-wave suppression. The coupling between $f_2(1270)$ and B^*B^* has not been obtained from any measured reaction channels yet; thus, in present work, we treat this coupling constant as a free parameter to be fixed later. The coupling constants between the scalar mesons and the final $\pi^+\pi^-$ are $g_{\sigma\pi\pi} = 16.2 \text{ GeV}^{-1}$ and $g_{f_0\pi\pi} = 2.40 \text{ GeV}^{-1}$, which are determined by fitting the corresponding partial widths.

In our model, besides the phase angles $\phi_R^{(n)}$ between the re-scattering processes and the direct two-pion emission, just as indicated above, two more parameters \mathcal{A} and κ are introduced for accounting the contribution from the direct process as shown in Eq. (2). For calculating the re-scattering amplitudes, a form factor is employed to describe the off-shell effects of the exchanged mesons. In the calculations, the form factor takes the monopole form, i.e., $\mathcal{F}(q^2) = (\Lambda^2 - m_E^2)/(q^2 - m_E^2)$, where m_E is the mass of the exchanged $B^{(*)}$ meson in the $B^{(*)}\bar{B}^{(*)} \rightarrow Y(nS)S, Y(nS)f_2$ transitions shown in Fig. 2, and Λ is usually reparameterized as $\Lambda = m_E + \alpha\Lambda_{QCD}$. It is worth pointing out that such an adoption has a certain arbitrariness, but the value Λ , which manifests all the unknown information about the non-perturbative QCD effects and the inner structure of the involved mesons, is determined by fitting data of various reactions; thus, it is believed that its value must fall in a reasonable range. Thus the arbitrariness is relatively alleviated. In present work, for $Y(5S) \rightarrow Y(nS)\pi^+\pi^-$ ($n = 1, 2$), $\alpha = 2$ is adopted. The coefficients of the relevant Lorentz structures in Eqs. (12)-(13) $g_{0S}^{(n)}, g_{0D}^{(n)}, g_{2S}^{(n)}, g_{2D}^{(n)}$ ($i = 1, 2, 3$) and $g_{2G}^{(n)}$ are determined by calculating the hadronic loops.

The $m_{\pi^+\pi^-}$ and $\cos\theta$ distributions measured by the Belle Collaboration as well as the partial decay widths $\Gamma_{Y(5S) \rightarrow Y(1S)\pi^+\pi^-} = 0.59 \pm 0.04 \pm 0.09 \text{ MeV}$ and $\Gamma_{Y(5S) \rightarrow Y(2S)\pi^+\pi^-} = 0.89 \pm 0.07 \pm 0.16 \text{ MeV}$, are taken as inputs to our work. All other input parameters, including the masses and widths of the involved particles, are listed in Table I. With the help of the MINUIT package, we fit the Belle data of $Y(5S) \rightarrow Y(1S, 2S)\pi^+\pi^-$ with the corresponding parameters being fixed and listed in Tables II and III.

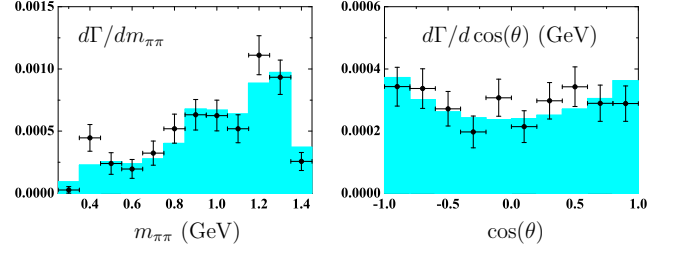


FIG. 3: (color online). Dipion invariant mass ($m_{\pi^+\pi^-}$) distribution (left-hand side) and the $\cos\theta$ distribution (right-hand side) measured by Belle [5] for the final state $Y(1S)\pi^+\pi^-$ (dots with error bars). The histograms are the best fit from our model.

TABLE II: The parameters for $Y(5S) \rightarrow Y(1S)\pi^+\pi^-$ that are gained by fitting the Belle data. Here $g_{f_2} = g_{f_2B^*B^*}g_{f_2\pi\pi}$.

| Parameter | Value | Parameter | Value (Rad) |
|-----------|---------------------|----------------------------|--------------------|
| F | 0.186 ± 0.061 | $\phi_{\sigma(600)}^{(1)}$ | -2.638 ± 0.735 |
| κ | 0.459 ± 0.084 | $\phi_{f_0(980)}^{(1)}$ | 1.539 ± 0.741 |
| g_{f_2} | 12.361 ± 20.109 | $\phi_{f_2(1270)}^{(1)}$ | -1.028 ± 2.050 |

The dipion invariant mass distribution $dG/dm_{\pi^+\pi^-}$ and the angular distribution $dG/d\cos\theta$ measured by the Belle Collaboration for $Y(5S) \rightarrow Y(1S)\pi^+\pi^-$ are shown in Fig. 3. The shaded histograms are the corresponding theoretical prediction by our model. The parameters for $Y(5S) \rightarrow Y(1S)\pi^+\pi^-$ are listed in Table II, yielding an integrated decay width of $\Gamma_{Y(5S) \rightarrow Y(1S)\pi^+\pi^-} = 0.54 \text{ MeV}$. The consistency between our results and the Belle data indicates that our model can naturally describe the anomalous production rate of $Y(1S)\pi^+\pi^-$ near the peak of $Y(5S)$ well, and, moreover, the predicted dipion invariant mass distribution and the $\cos\theta$ distribution also coincide with the data.

For $Y(5S) \rightarrow Y(2S)\pi^+\pi^-$, we carry out a similar calculation. The shaded histograms are our best fit to the experimental data, and the corresponding parameters are listed in Table III. The integrated decay width of $\Gamma_{Y(5S) \rightarrow Y(2S)\pi^+\pi^-} = 0.845 \text{ MeV}$. Our results also confirm that the contribution from $f_0(980)$ is rather small compared to the $Y(5S) \rightarrow Y(2S)\pi^+\pi^-$ process.

However, one notices a discrepancy. As shown in Fig. 4, the dipion invariant mass distribution of the $Y(2S)\pi^+\pi^-$ production near the peak of $Y(5S)$ is well reproduced by our model. However, applying the same fitting parameters, the predicted $dG/d\cos\theta$ of $Y(5S) \rightarrow Y(2S)\pi^+\pi^-$ (the histogram on the left panel of Fig. 4), displays a different behavior from the Belle data for the $Y(2S)\pi^+\pi^-$ channel (dots with error bars in the right-hand diagram of Fig. 4).

In summary, stimulated by the recent Belle observation of

TABLE III: The fitted parameters for $Y(5S) \rightarrow Y(2S)\pi^+\pi^-$.

| Parameter | Value | Parameter | Value (Rad) |
|-----------|-------------------|----------------------------|--------------------|
| F | 2.315 ± 1.904 | $\phi_{\sigma(600)}^{(2)}$ | -0.297 ± 0.567 |
| κ | 0.572 ± 0.283 | $\phi_{f_0(980)}^{(2)}$ | -3.140 ± 4.532 |

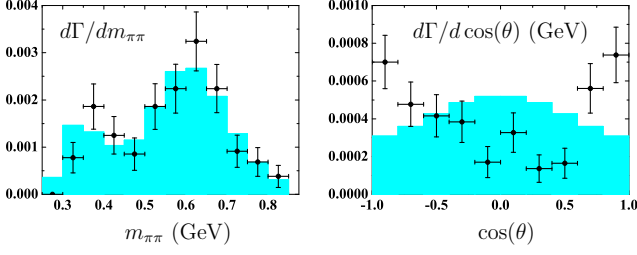


FIG. 4: (color online). The comparison between the fitting result (histogram) for $\Upsilon(5S) \rightarrow \Upsilon(2S)\pi^+\pi^-$ and the Belle data (dots with error bars) [5].

anomalously large production rates of $\Upsilon(1S, 2S)\pi^+\pi^-$ near the peak of $\Upsilon(10860)$, carefully studying the previous works along the line, we suggest that both the direct dipion emission process and the processes via intermediate physical states which are the so-called final state interactions, contribute to the amplitude, and their interference results in the observed dipion emission of $\Upsilon(10860)$. In our scenario, the inverse rates $\Gamma(\Upsilon(10860) \rightarrow \Upsilon(2S)\pi^+\pi^-) > \Gamma(\Upsilon(10860) \rightarrow \Upsilon(1S)\pi^+\pi^-)$ can also be naturally understood, i.e., their rates are determined by the interference between the contributions of the direct emission and the final interactions.

Fitting the decay rates of $\Upsilon(10860) \rightarrow \Upsilon(1S, 2S)\pi^+\pi^-$, we further theoretically investigate the dipion invariant mass and the $\cos\theta$ distributions of $\Upsilon(10860) \rightarrow \Upsilon(1S, 2S)\pi^+\pi^-$ and make a comparison with the Belle data. Indeed, it is observed that if the final state interactions overwhelmingly dominate the transitions $\Upsilon(10860) \rightarrow \Upsilon(1S) + \pi^+\pi^-$ and $\Upsilon(10860) \rightarrow \Upsilon(2S)\pi^+\pi^-$, the lineshapes of the differential widths over the dipion invariant mass and $\cos\theta$ cannot be well fitted. It indicates that the interference effect plays a crucial role for fully understanding the Belle observation [5].

What is more important is that our model demonstrated

in this paper shows that the tetraquark scenario proposed by Ali *et al.* [6–8] does not provide a satisfactory understanding of the anomalous $e^+e^- \rightarrow \psi(2S)\pi^+\pi^-$ production at $\sqrt{s} = 10.870$ GeV. However, one cannot rule out that there might be a fraction of the tetraquark component in $\Upsilon(10860)$ that also contributes to the dipion transition. But, so far, it seems that a contribution from such an exotic state is not necessary for just understanding the Belle data. Instead, our study presented in this paper indicates that the Belle observation can be naturally explained by the interference between the direct dipion emission and the final state interactions.

It is worth pointing out that in our model, our theoretical predictions on both anomalous production rates of $\Gamma(\Upsilon(10860) \rightarrow \Upsilon(2S)\pi^+\pi^-)$ and $\Gamma(\Upsilon(10860) \rightarrow \Upsilon(1S)\pi^+\pi^-)$ coincide with the data of the Belle collaboration, and also satisfactorily describe the dipion invariant mass and the $\cos\theta$ distributions of $\Upsilon(10860) \rightarrow \Upsilon(1S)\pi^+\pi^-$ as well as the dipion invariant mass distribution of $\Upsilon(10860) \rightarrow \Upsilon(2S)\pi^+\pi^-$. However, a new and intriguing puzzle is proposed since the predicted $d\Gamma/d\cos\theta$ of $\Upsilon(10860) \rightarrow \Upsilon(2S)\pi^+\pi^-$ is inverse to the Belle data just presented in the right panels of Figs. 1 and 4. Associated with further theoretical exploration, future experimental study from Belle-II and SuperB will be helpful to clarify this new puzzle and give a definite conclusion.

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